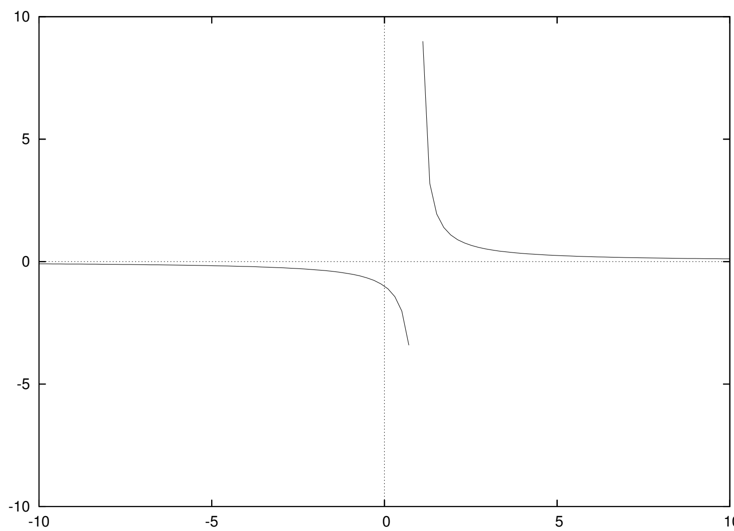
Name ____________________ Student Number ____________

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

(1) Find all horizontal and vertical asympotes of $f(x) = \dfrac{x}{x^2 - x}$. Sketch the graph of $f(x)$.

$f(x)$ is a rational function so the only place that vertical asymptotes will occur is when $x^2 - x = 0$. This is $x = 0, 1$. However, $\lim_{x\to 0} f(x) = -1$ (you should verify this), so that $x = 0$ is *not* a vertical asymptote. Verify that $\lim_{x\to 1^-} f(x) = -\infty$ and $\lim_{x\to 1^+} f(x) = \infty$ so that $x = 1$ is a vertical asymptote.

For a horizontal asymptote, $\lim_{x\to\pm\infty} f(x) = 0$, so $y = 0$ is a horizontal asymptote.



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(2) Evaluate the following limits:

(a) $\displaystyle\lim_{x\to 1} \frac{x^2+x-2}{x^2-1}$

$$= \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x+2}{x+1} = \frac{3}{2}$$

(b) $\displaystyle\lim_{x\to -1} \frac{x-1}{|x-1|}$

$$= \frac{-1-1}{|-1-1|} = \frac{-2}{|-2|} = \frac{-2}{2} = -1$$

(3) Remove the discontinuity from the following function:

$$f(x) = \begin{cases} \frac{x^3+x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

$f(x)$ is discontinuous at $x = 0$ since $\lim_{x\to 0} f(x) = 1$ (verify) and $f(0) = 2$. Invent the new function

$$g(x) = \begin{cases} f(x), & x \neq 0 \\ 1, & x = 1 \end{cases}$$

which is the same as $g(x) = x^2 + 1$.